



**CENTRAL
UNIVERSITY**

END OF FIRST SEMESTER EXAMINATION: 2017/2018

FACULTY OF ARTS AND SOCIAL SCIENCES

DEPARTMENT OF ECONOMICS

ECON 204 (3 CREDITS)

MATHEMATICS FOR ECONOMIST II

LEVEL 200

MAY 2018

2 HOURS

STUDENT ID No.....

INSTRUCTIONS

ANSWER ALL QUESTIONS

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THE INVIGILATOR**

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SECTION A
Answer all the questions

Question 1

Differentiate the following:

(a) $y = x^3 e^{2x}$

(b) $y = \ln(x^2 + 2x + 1)$

(c) $y = a^{1-2x}$

(d) $y = \ln(x(x + 1)^4)$

(e) $f(x) = \log_a(2x^2 + 1)$

Question 2

Find ALL the first and the second partial derivatives for each of the following functions

1. $z = e^{x^2+y^2}$

2. $z = e^{2x^2+3y}$

3. $z = \ln(7x + 2y)$

4. $z = \ln(x^2 + 4y^2)$

Question 3

Given the function $z = \ln(2x^2 - 12x + y^2 - 10y)$

- i. Find the value of the critical values and
- ii. Indicate whether the function is at maximum or minimum

Question 4

- a) Optimize the following functions subject to the given constraint and estimate the effect on the value of the objective function from a 1-unit change in the constant of the constraint

i. $z = 4x^2 - 2xy + 6y^2$ subject to $x + y = 72$

ii. $f(x, y, z) = 4xyz^2$ subject to $x + y + z = 56$

- b) What combination of goods x and y should a firm produce to minimize costs when the joint cost function is $c = 6x^2 + 10y^2 - xy + 30$ and the firm has a production quota of $x + y = 34$? Estimate the effect on the cost if the production quota is reduced by 1-unit.

Question 5

- a) Use the *Jacobian* to test for functional dependence in the following system of equations:

$$y_1 = 6x_1 + 4x_2$$

$$y_2 = 7x_1 + 9x_2$$

- b) A firm produces two goods in pure competition and has the following total revenue and total cost functions:

$$TR = 15Q_1 + 18Q_2$$

$$TC = 2Q_1^2 + 2Q_1Q_2 + 3Q_2^2$$

The two goods are *technically related* in production, since the marginal cost of one is dependent on the level of output of the other. Maximize profits for the firm, using:

- i) *Cramer's rule* for the first-order condition and
- ii) *The Hessian* for the second-order condition