CENTRAL UNIVERSITY

FAITH · INTEGRITY · EXCELLENCE

END OF FIRST SEMESTER 2020/2021 RESIT EXAMINATIONS

FACULTY OF ARTS AND SOCIAL SCIENCES

DEPARTMENT OF ECONOMICS

ECON 203: MATHEMATICS FOR ECONOMISTS I

(LEVEL 200)

Instructions:

Answer all questions in Section A and any other three questions in Section B.

Time Allowed: **2 Hours**

Name of Examiner: MR. DANIEL OFFEI

SECTION A

Indicate whether the following statement is *True* of *False*. (10 marks)

1. A function f(x) is said to be increasing at $x = a$ if in the immediate vicinity of the point (a, f(a)) the graph of the function falls as it moves from left to right. **TRUE/ FALSE**

2. If at a point "b" of a function f(x) the first derivative evaluated at the point "b" is greater than zero (ie. $f'(b) > 0$), then we can conclude that the function is increasing at $x = b$.
   TRUE/ FALSE

3. If at a point "b" of a function f(x) the first derivative evaluated at the point "b" is less than zero, (ie. $f'(b) < 0$) then we can conclude that the function is decreasing at $x = b$.

TRUE/ FALSE

4. If at the point "a" on the function f(x), the evaluation of the point "a" on the second derivative of the function is greater than zero, (ie. $f''(a) > 0$), then f(x) is concave at $x =$

   TRUE/ FALSE

5. If at the point "a" on the function f(x), the evaluation of the point "a" on the second derivative of the function is less than zero, (ie. $f''(a) < 0$) then f(x) is convex at $x = a$.

   TRUE/FALSE

SECTION B

Answer QUESTION ONE and any other TWO questions

a) If the demand function is $P = 100 - 3Q$;
   i.   Find the expression for Total Revenue (TR) in terms of Q.
   ii.   Find the value of marginal revenue(MR) at Q=15

b) If the Total Revenue (TR) function of a good is given by $1000Q - 4Q^2$. If the current demand is 40, find the approximate change in the value of TR due to;
   i. a 2 unit increase in Q
   ii.   a 3 unit decrease in Q

c) The demand equation of a good is given by $P + 2Q = 20$ and the total cost function is $C = Q^3 - 8Q^2 + 20Q + 2$, find;
   i.   the level of output that maximises total revenue
   ii.   the maximum profit and the value of Q at which it is achieved. Verify that at this value of Q, MR=MC.

d) Given the utility function $U = X_1^{\frac{1}{2}} X_2^{\frac{1}{2}}$,
   i.   Find the expression for Marginal Rate for Commodity Substitution (MRCS) in terms of $X_1 \, and \, X_2$.

   ii.   Calculate a particular value of MRCS for the indifference curve that passes through the point (120, 300), hence estimate the increase in X_2 required to maintain the current level of utility when X_1 decreases by 4 units.

e) Given the production function as $Q = K^2 + 2L^2$, write down the expression for marginal products of capital and labour and show that;

i.   Marginal Rate of Technical Substitution (MRTS) $= 2LK^{-1}$

ii. $K\frac{\partial Q}{\partial K} + L\frac{\partial Q}{\partial L} = 2Q$

(40 marks)

QUESTION TWO

A firm's short-run production function is given by

$$Q = \frac{6}{L^{-2}} - 0.2L^3$$, where L denotes the number of workers and Q is the
Output.

i.   Find the size of the workforce that maximize output and find the maximum output level that can be achieved with that size of the workforce.

ii.   Find the size of the workforce that maximizes Average Product of Labour (AP_L). Calculate Marginal Product of labour (MP_L) and Average Product of Labour (AP_L) at this value of L
(25 marks)

QUESTION THREE
An individual utility function is given by

$$U = 100x_1 + 45x_2 + 6x_1x_2 - 2x_1^2 - x_2^2$$

Where x_1, the amount of leisure measured in hours per week and x_2 is earned income measured in Ghana Cedis (Ghs) per week.

a) Determine the level of Marginal Utility when $x_1 = 138$ and $x_2 = 500$
b) Based on your answer in (a), estimate the change in Total Utility (U) if the individual works for an extra 2 hours, which increases earned income by Ghs 10 per week.
c) Does the law of diminishing marginal utility holds for this function?

(25 marks)

QUESTION FOUR

Given the demand function for commodity B as $Q_B = 100 - 4P - P_A + 0.2Y^2$, where P, P_A and Y are Price of Commodity B, Price of related good A and Income of consumers respectively.

If P = 10, P_A = 15 and Y = 100, find

(a) the price elasticity of demand and state the nature of its elasticity. What advice will you give to your company with that nature of elasticity in order to increase profit?

(b) the cross-price elasticity of demand and state the relationship between commodity A and B.

(c) the income elasticity of demand and indicate whether commodity B is normal of inferior.
(25 marks)

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