Journal of Hydrology, 79 (1985) 95–106
Elsevier Science Publishers B.V., Amsterdam — Printed in The Netherlands

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SCALING THE EXPONENTIAL SOIL-WATER DIFFUSIVITY FOR SOILS FROM GHANA

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(Received September 18, 1984; accepted for publication January 31, 1985)

ABSTRACT

Laryea, K.B. and Abenney-Mickson, S., 1985. Scaling the exponential soil-water diffusivity for soils from Ghana. J. Hydrol., 79: 95–106.

Horizontal infiltration experiments were conducted on laboratory packed columns of six tropical soils chosen from two different ecological zones of Ghana, namely, the coastal savannah and the forest regions. Soil-water diffusivities of these soils calculated using the method of Bruce and Klute (1956) were then fitted with an exponential regression equation to determine the values of the "Universal" constants, α, γ and β, proposed by Reichardt et al. (1972), Miller and Bresler (1977) and Brutsaert (1979). Values obtained for α, γ and β for these soils did not agree with those suggested by Reichardt et al. and Brutsaert thus negating the universality concept proposed by these authors.

INTRODUCTION

The one-dimensional non-hysteretic horizontal movement of water in unsaturated soils can be described by the nonlinear diffusion equation:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left[D(\theta)\frac{\partial \theta}{\partial x}\right] \tag{1}$$

where θ is the volumetric water content ($m^3\ m^{-3}$); $D(\theta)$ is the soil-water diffusivity ($m^2\ s^{-1}$); x is the space coordinate (m); and t is time (s). In a numerical solution of eqn. (1), Gardner and Mayhugh (1958) presented an exponential equation of the form:

$$D(\theta) = D_n(\theta_n)\exp(\beta\Theta) \tag{2}$$

which was used by Reichardt et al. (1972) to scale the data of sorption experiments conducted on eight different air-dry soils. These workers presented their results by a single regression equation involving dimensionless variables.

In eqn. (2), $D_n(\theta_n)$ is the diffusivity at the initial (reference) water content, θ_n; θ_o is the water content at satiation, β is a constant and the reduced water content is given by:

0022-1694/85/$03.30

$$\Theta = (\theta - \theta_n)/(\theta_o - \theta_n) \tag{3}$$

In reconsidering the analysis of Reichardt et al. (1972), Miller and Bresler (1977) have expressed eqn. (2) by a simple regression equation of the form:

$$D(\theta) = \alpha\phi_f^2 \exp(\beta\Theta) \tag{4}$$

where α and β are constants and ϕ_f is the Boltzmann variable at the wetting front defined by:

$$\phi_f = x_f/t^{1/2} \tag{5}$$

in which x_f is the distance to the wetting front from $x = 0$ where the water content is maintained at $\theta = \theta_o$ and where water infiltrates horizontally into a soil of an initially air-dry water content θ_n. The analysis of Miller and Bresler (1977) showed that α and β may be considered universal constants of values 10^{-3} and 8 respectively. Clothier and White (1981) have provided an exception to this, thus negating the universality concept.

In a recent theoretical analysis Brutsaert (1979) has shown that α and β are mutually dependent and that only one constant is needed. Furthermore, Brutsaert (1979) showed that the exponential form of soil-water diffusivity can be scaled with the sorptivity, S, defined as:

$$S = \int_{\theta_n}^{\theta_o} \phi \mathrm{d}\theta = I/t^{1/2} \tag{6}$$

In eqn. (6), I is the cumulative infiltrated water volume. In terms of the sorptivity, the following equation was proposed by Brutsaert (1979):

$$D(\Theta) = \gamma[S/(\theta_o - \theta_n)]^2 \exp(\beta\Theta) \tag{7}$$

where the constant γ is 1.44×10^{-3} when the recommended value of 8 for β is used (Reichardt et al. 1972).

It is the purpose of this paper to analyse sorption experiments conducted on six tropical soils using the wetting-front parameter proposed by Miller and Bresler (1977) as well as the sorptivity parameter given by Brutsaert (1979). This leads to the following dimensionless forms for the soil moisture diffusivity:

$$D''(\theta) = D(\theta)/\phi_f^2 \tag{4A}$$

and

$$D^*(\theta) = D(\theta)\cdot[S/(\theta_o - \theta_n)]^{-2} \tag{7A}$$

for which the following relations will be tested:

$$D''(\theta) = \alpha \exp(\beta\Theta) \tag{4B}$$

and

$$D^*(\theta) = \gamma \exp(\beta\Theta) \tag{7B}$$

The parameters α and γ are related via the shape of the wetting profile (θ against x):

$$I = \int_0^{x_f} (\theta - \theta_n)\mathrm{d}x = \delta \cdot (\theta_o - \theta_n) \cdot x_f$$

where δ is the shape factor for the wetting profile. Its value is 0.5 for a linear dependence of θ on x, and 1.0 for a rectangular shape with a sharp wetting front. It may be expected that δ will be slightly below 1.0 for horizontal infiltration into initially dry soils. This comparison leads to:

$$S/(\theta_o - \theta_n) = \delta \cdot \phi_f$$

and $D''/D^* = \alpha/\gamma = \delta^2$ or $\delta = (\alpha/\gamma)^{1/2}$.

MATERIALS AND METHODS

The experiments were conducted using six top soils (0—15 cm) of known series selected from two main ecological zones of Ghana, namely: the coastal savannah and the forest zones. A description of the soils is given in Table 1. A large well-mixed sample of each soil was sieved through a 0.25 mm sieve.

The infiltration column used in this study is similar to that used by Elrick et al. (1979). Uniformity of packing of the column of soil was achieved by pouring the soil into the column evenly and tapping at the same time as the soil was being poured. The infiltration solution, in all experiments was 0.01 N $CaSO_4$. In all the experiments the temperature of the infiltration solution was 30 ± 1°C. The cumulative volume of solution as well as the distance to the wetting front was recorded as a function of the square root of time as an initial check on the preservation of similarity with regards to water flow. Data were obtained from experiments terminated at different elapsed times.

At the end of each sorption experiment the time was recorded, the column sliced into sections and the soil in each section (of known volume) was used to calculate the bulk density, the gravimetric and volumetric water contents. The soil-water diffusivity function $D(\theta)$ was then calculated using the volumetric water content distribution and an equation of the form (Bruce and Klute, 1956):

$$D(\theta) = -\tfrac{1}{2} \frac{\mathrm{d}\phi}{\mathrm{d}\theta} \int_{\theta_n}^{\theta} \phi \mathrm{d}\theta \qquad (8)$$

RESULTS AND DISCUSSION

Figure 1 presents the water content distribution of the six soils for the sorption experiments listed in Table 2.

The smoothed curves of best fit (solid lines) were drawn by eye. The

TABLE 1

Description and some properties of the soils

Soil number and series name	Great soil group	Parent material	Vegetation	pH in 0.01 N $CaCl_2$	Organic carbon (%)	CEC (meq/100 g)	Particle-size analysis		
							% sand	% silt	% clay
1. Abenia	Forest Oxysol	biotite granite schist	forest	4.14	1.00	11.92	65.40	6.11	28.49
2. Adentan	Savannah Ochrosol	acid gneiss	tall grass savannah	5.18	0.36	4.50	77.14	11.50	11.36
3. Boi	Forest Oxysol	phyllite	forest	4.20	0.96	11.48	57.28	2.04	40.68
4. Nyigbenya	Savvanah Ochrosol	acid gneiss schist	tall grass savannah	5.14	0.71	7.30	81.78	10.12	8.10
5. Tikobo	Forest Oxysol	tertiary sand	forest	4.64	0.77	6.16	87.84	4.05	8.11
6. Toje	Savannah Ochrosol	tertiary sand	tall grass savannah	5.20	0.24	4.70	91.97	4.01	4.02

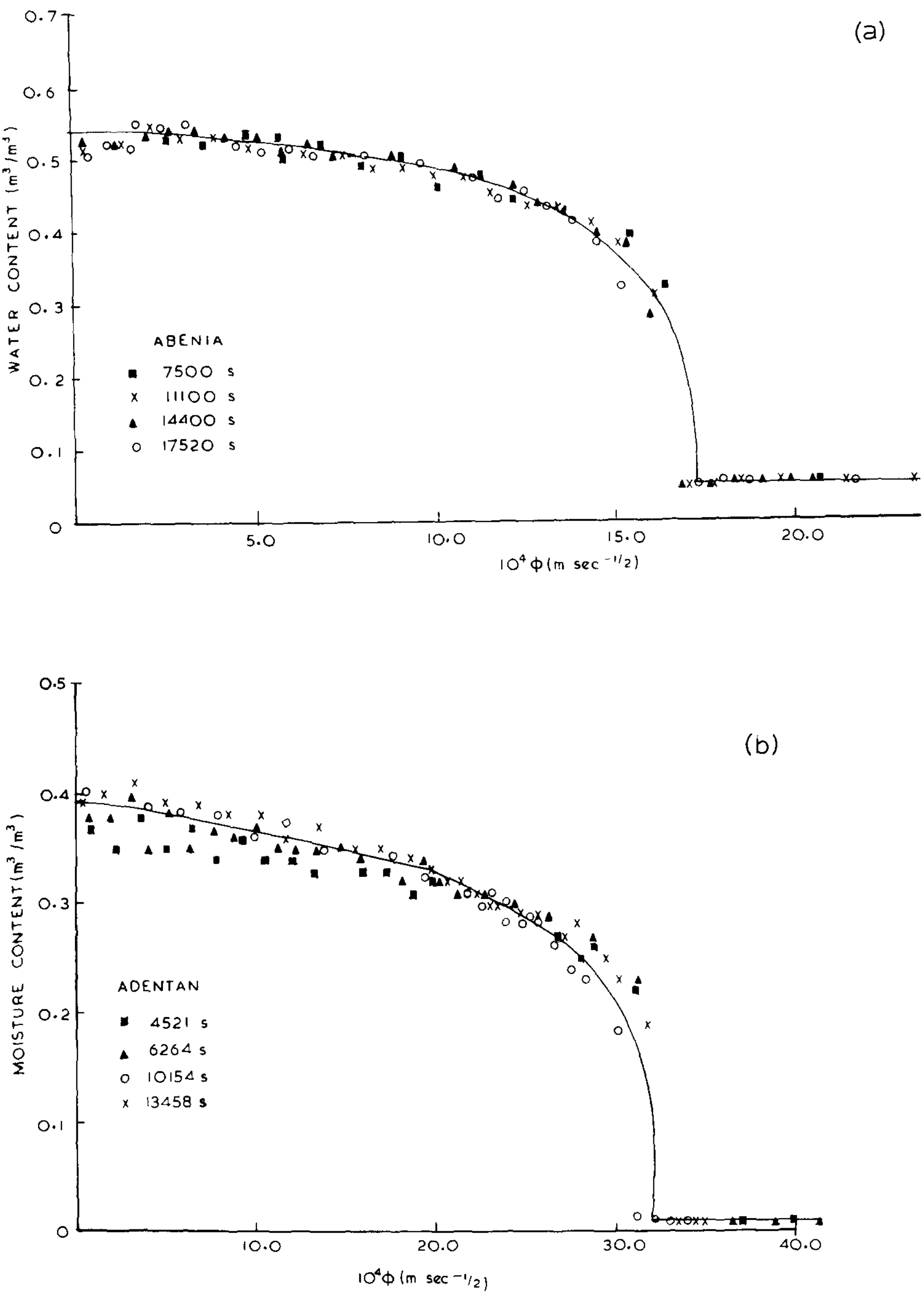
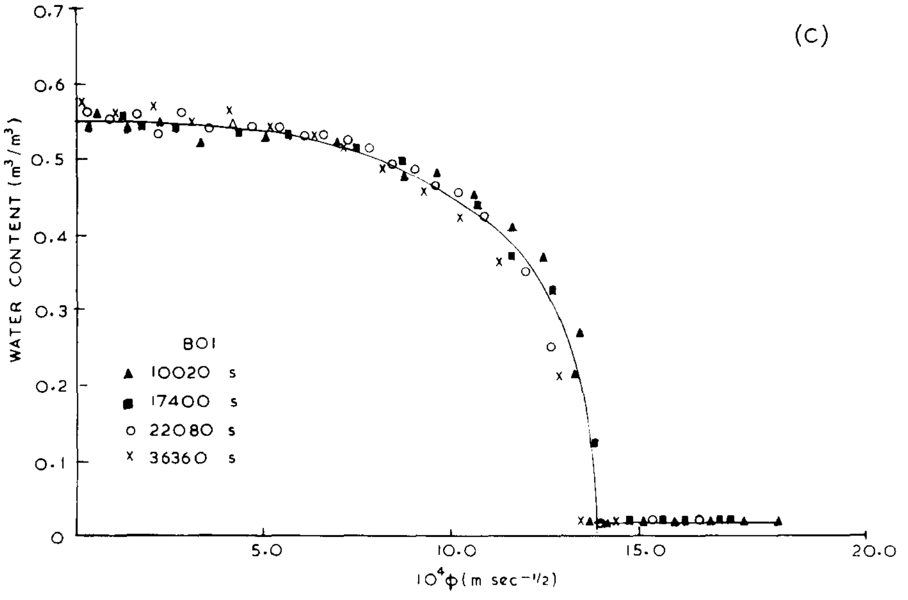


Fig. 1. Experimental water-content distribution of the sorption experiments listed in Table 2.



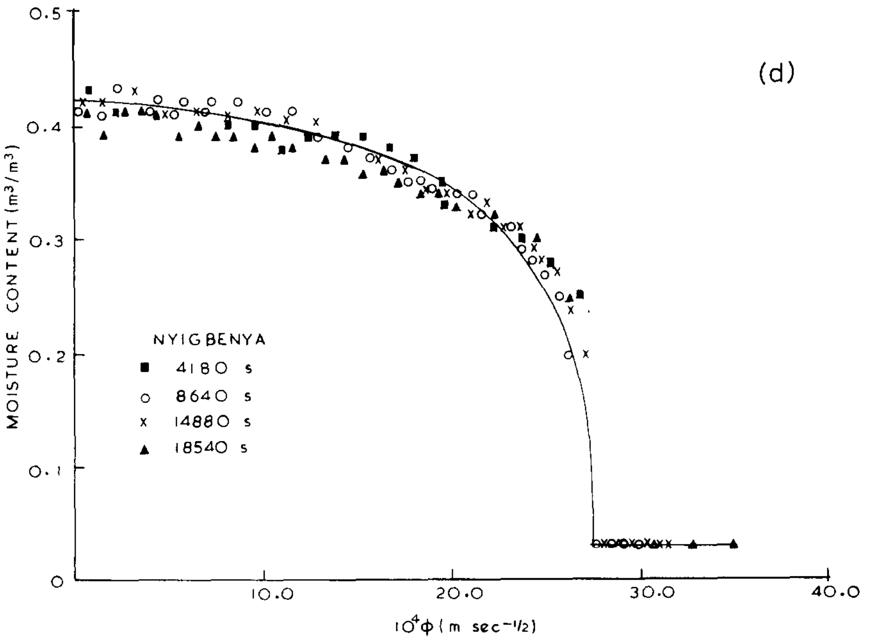
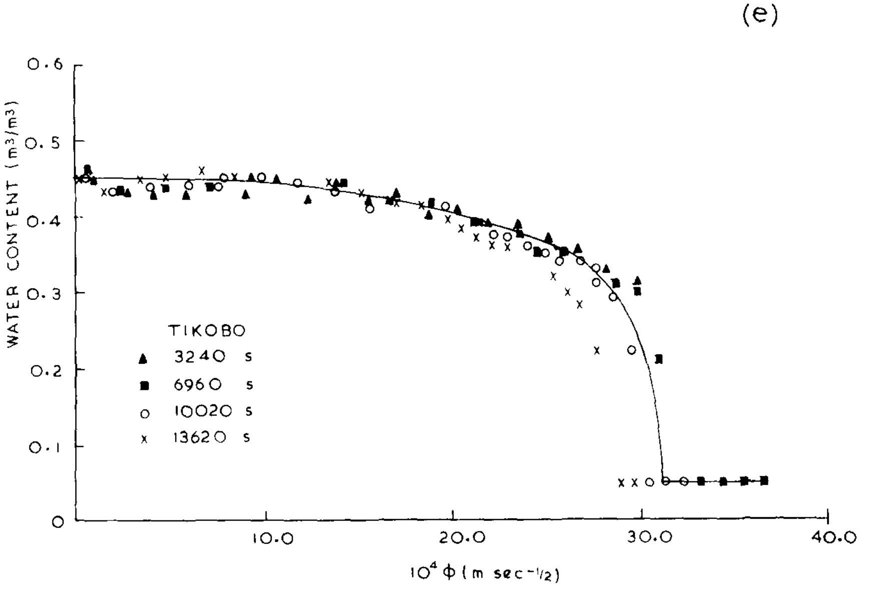


Fig. 1. (*Continued*)



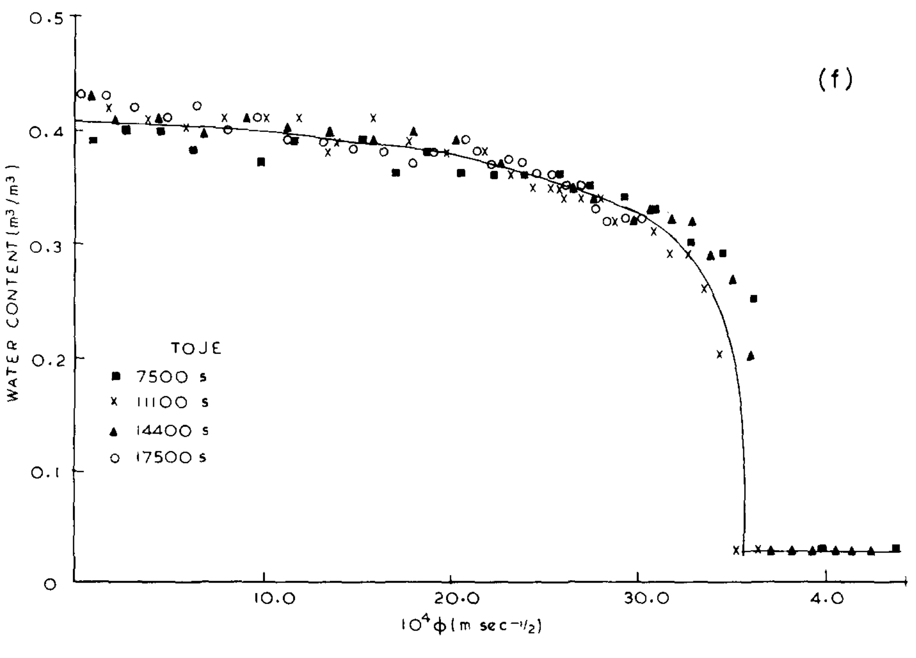


Fig. 1. (*Continued*)

TABLE 2

Summary of the experimental conditions imposed during sorption

Soil	Experimental set	Initial volumetric water content θ_n ($m^3\ m^{-3}$)	Water content at satiation θ_o ($m^3\ m^{-3}$)	Dry bulk density ρ_b ($Mg\ m^{-3}$)	Measured sorptivity $10^4 S$ ($m\ s^{-1/2}$)	Measured Boltzmann variable at wetting front $10^3 \phi_f$ ($m\ s^{-1/2}$)	Elapsed time $10^{-4} t$ (s)
Abenia	A	0.05	0.52	1.04	7.135	1.630	1.110
	B	0.05	0.52	1.03	6.822	1.536	1.752
	C	0.04	0.53	1.03	6.892	1.579	1.440
	D	0.05	0.53	1.03	7.430	1.690	0.750
Adentan	A	0.01	0.39	1.42	10.020	3.147	1.346
	B	0.01	0.40	1.40	9.598	3.035	1.016
	C	0.01	0.38	1.40	9.926	3.069	0.627
	D	0.01	0.37	1.43	9.401	3.049	0.452
Boi	A	0.02	0.57	0.98	5.866	1.225	3.636
	B	0.02	0.56	0.99	6.106	1.274	2.208
	C	0.02	0.56	1.01	6.393	1.367	1.002
	D	0.02	0.56	1.02	6.195	1.300	1.740
Nyigbenya	A	0.03	0.42	1.34	9.395	2.705	1.488
	B	0.03	0.41	1.39	9.097	2.622	1.854
	C	0.03	0.41	1.37	8.872	2.627	0.864
	D	0.03	0.43	1.35	10.300	2.991	0.408
Tikobo	A	0.05	0.45	1.26	10.160	2.788	1.362
	B	0.05	0.45	1.27	10.670	2.958	1.002
	C	0.05	0.45	1.26	11.430	3.173	0.324
	D	0.04	0.46	1.27	10.920	3.088	0.696
Toje	A	0.03	0.44	1.35	12.190	3.489	0.978
	B	0.03	0.43	1.33	10.520	3.111	1.440
	C	0.03	0.43	1.36	12.700	3.622	0.756
	D	0.03	0.40	1.35	12.940	3.747	0.270

water content distribution of all the six soils preserved similarity reasonably well.

Variability in packing of the column, swelling of the soil on wetting and problems of air entrapment likely account for the variability in the data. Both the cumulative volume of solution as well as the distance to the wetting front as functions of $t^{1/2}$ gave excellent straight lines with correlation coefficients ranging from 0.98 to 0.99. Because of the experimental difficulties referred to above, the slopes of these lines (i.e. the measured S and ϕ_f) differed slightly from one column to another as observed in Table 2.

Data for the soil-water diffusivity, $D(\theta)$, calculated using eqn. (8) and Fig. 1 are presented in Fig. 2 together with the straight lines approximating their exponential behaviour.

These $D(\theta)$ data were divided by either ϕ_f^2 (in Table 2) or $[S/(\theta_0 - \theta_n)]^2$ and plotted against the reduced water content (Θ; Figs. 3 and 4) in order to calculate α, β and γ in eqns. (4) and (7). The correlation coefficient of 0.98 and 0.99 for the plots presented in Figs. 3 and 4, respectively, are significant

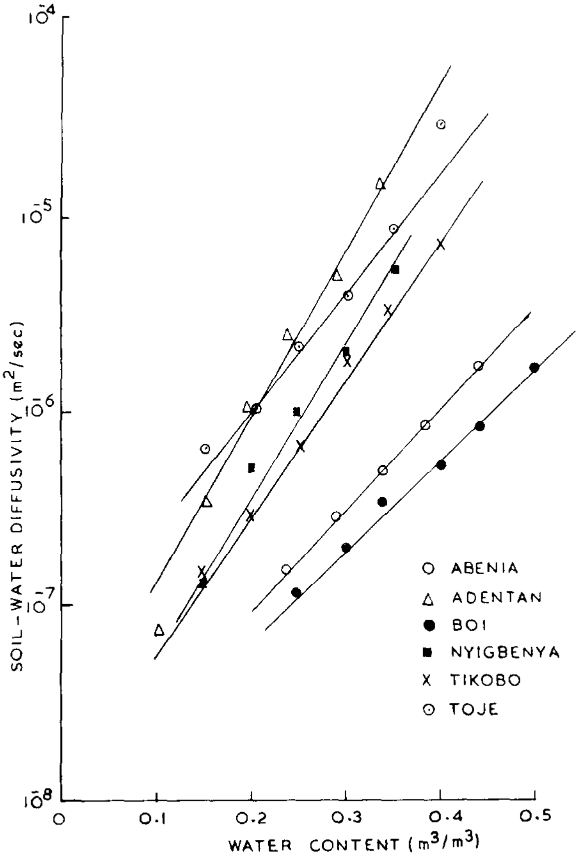


Fig. 2. The $D(\theta)$ relationships for the soils.

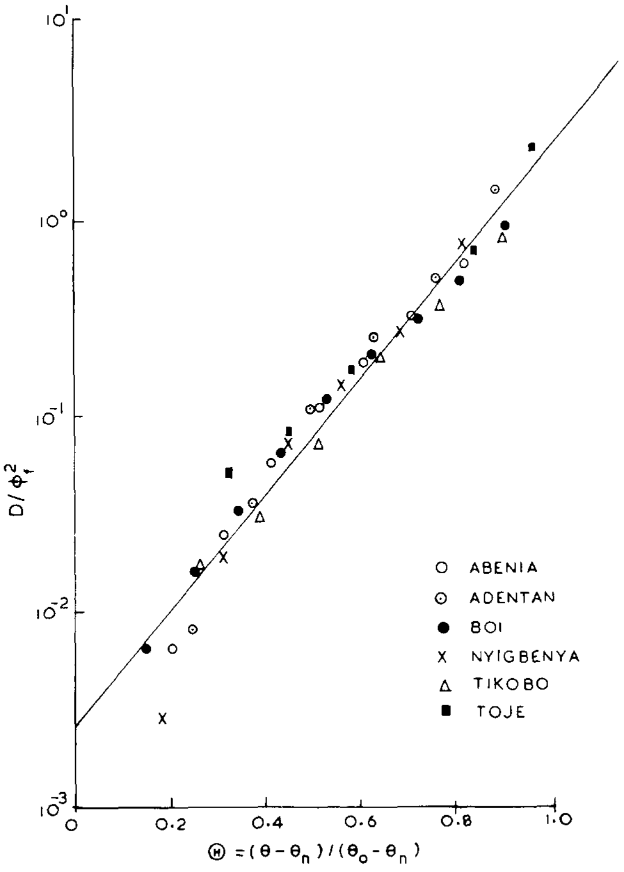


Fig. 3. The plot of $D(\theta)/\phi_f$ versus reduced water content Θ.

at the 1% level. This indicates that both scaling procedures are applicable to these tropical soils.

From Fig. 3, β was found to be 6.84 and $\alpha \doteq 2.60 \times 10^{-3}$. The value of γ was found from Fig. 4 to be 3.04×10^{-3}. The shape factor for the wetting profile is $\delta = \sqrt{\alpha/\gamma} = 0.92$.

Using eqns. 11 and 16 of Brutsaert (1979) with $\beta = 7$ and the confluent hypergeometric function tabulated by Abramowitz and Stegun (1964, pp. 516—535), which gives $M(-0.5, 0.5, 7) = -107.780$, α and γ values obtained were 2.44×10^{-3} and 3.46×10^{-3} respectively. These values agree fairly well with those calculated from the regression equation of Fig. 3 and 4. Note that $\beta = 6.84$ was approximated to 7 for the calculation of the confluent hypergeometric function and that possibly accounts for the difference of 1.6×10^{-4} between the calculated α values and that from the regression equation of Fig. 3. It may also account for the difference of 4.2×10^{-4} between the γ value calculated with eqn. 16 of Brutsaert (1979) and that from the regression equation of Fig. 4.

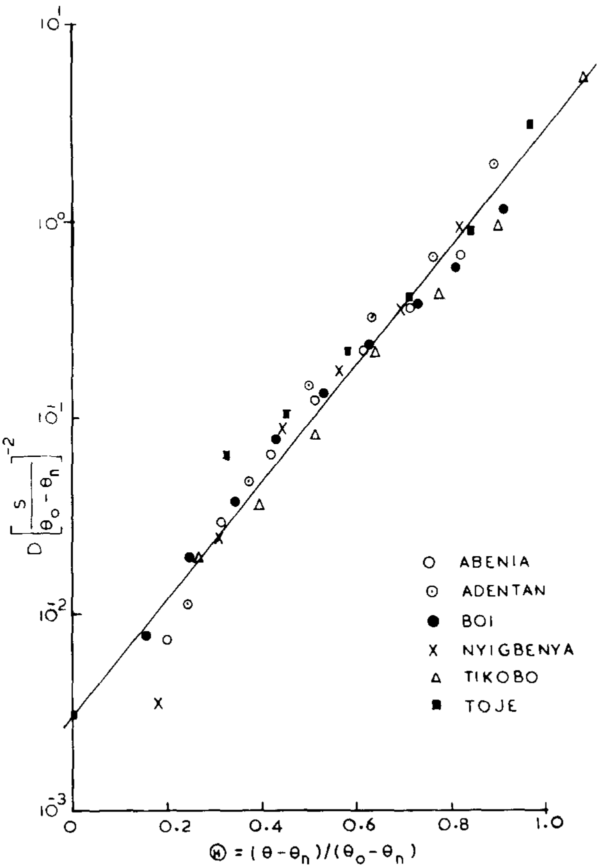


Fig. 4. The plot of $D(\theta)\ [S/(\theta_0 - \theta_n)]^{-2}$ versus the reduced water content Θ.

It is pertinent to point out that the values of $\alpha = 2.44 \times 10^{-3}$ and $\beta = 6.84$ negate the universality concept of α being 1×10^{-3} and $\beta = 8$. An exception to this universality concept was also reported by Clothier and White (1981) while Miller and Bresler (1977) found that depending on the Sodium Absorption Ratio (SAR) of the soil α value may range from 0.95×10^{-3} to 12×10^{-3} and β from 5.2 to 7.4.

The method of scaling the soil-water diffusivity suggested by Miller and Bresler (1977) and used in this study is related to the scaling of the soil-water diffusivity by the method of Reichardt et al. (1972) as follows:

$$D'' = kD' \tag{9}$$

where $k(= \sigma\lambda_s/\phi_s^2\eta)$ is a constant and its value depends on the surface tension σ, viscosity η and the Boltzmann transform ϕ_s for the soil chosen as the standard. In eqn. (9), $D''[= D(\theta)/\phi_f^2]$ is the dimensionless soil-water diffusivity suggested by Miller and Bresler (1977) and $D'[= \eta D(\theta)/\lambda_i\sigma]$ is the scaled soil-water diffusivity used by Reichardt et al. (1972), with λ_i

being the characteristic length of a particular soil calculated using the relationship:

$$\lambda_i/\lambda_s = (\phi_i/\phi_s)^2 \qquad (10)$$

Here $\phi_i(= x/t^{1/2})$ is the Boltzmann transform for a particular soil i.

The value of λ_s, the characteristic length of the standard soil is assumed to be unity and does not therefore affect the value of k. In this study k was found to be 5.33×10^5.

CONCLUSION

It is concluded from the preceding discussion that the scaling procedures suggested in the literature are also applicable to soils from Ghana. But that the constants α, β and γ vary according to the type of soil, sodium absorption ratio (as suggested by Miller and Bresler, 1977) and the scaling technique used to obtain the dimensionless soil-water diffusivity. The relationship between the method of scaling of Reichardt et al. (1972) and that of Miller and Bresler (1977) depends on the Boltzmann transform of the soil selected as the standard, the surface tension and the viscosity of the infiltrating solution. The shape factor δ for the wetting profile appears to be around 0.90.

ACKNOWLEDGEMENT

The authors are grateful to Dr. D.K. Acquaye for providing some of the data used in Table 1 to characterise the forest soil.

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